INdAM Meeting:
Hyperbolic Dynamical Systems in the Sciences

Corinaldo (Italy)
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ABSTRACTS

Viviane Baladi (Ecole Normale Supérieure, Paris)
Anisotropic Sobolev spaces adapted to piecewise hyperbolic dynamics

Strong ergodic properties (such as exponential mixing) have been proved for various smooth dynamical systems by first obtaining a spectral gap for a suitable "transfer" operator acting on an appropriate Banach space. Some natural dynamical systems, such as discrete or continuous-time billiards, are only piecewise smooth, and the discontinuities pose serious technical problems in the construction of the Banach norm. With Sebastien Gouezel (J. Mod. Dyn. 2010), we recently showed that classical tools such as complex interpolation on anisotropic Sobolev-Triebel spaces, and an old result of Strichartz on Fourier multipliers, can solve those problems, under a bunching condition. (The bunching condition replaces a smoothness assumption on the stable bundles which was necessary in a previous work.) I will also very briefly mention work in progress with Balint and Gouezel on the one hand, and Liverani on the other hand, the ultimate goal of which is to establish exponential mixing for continuous-time 2-d Sinai billiards.

Péter Bálint (Budapest University of Technology and Economics)
Dispersing billiards with cusps and tunnels

Two dimensional dispersing billiards with finite horizon and disjoint scatterers are among the best known examples of strongly chaotic dynamical systems. If, however, two scatterers of the configuration touch tangentially, forming a cusp, the billiard particle may experience arbitrary long sequences of successive collisions in the resulting region, which may slow down the convergence to equilibrium. The situation when the two scatterers do not actually touch, their distance is, however, some small ε, may be referred to as a billiard with a tunnel. In this talk I would like to summarize what is known about the statistical properties of dispersing billiards with cusps, and I am planning to describe some work in progress on dispersing billiards with tunnels.
(Joint work with N. Chernov and D. Dolgopyat)

EMANUELE CAGLIOTI (Sapienza Università di Roma)
FRANCESCA TRIA (ISI Foundation, Torino)

Distance-based methods for phylogenetic reconstruction and applications in linguistics and biology

Phylogenetic reconstruction belongs to a general class of inverse problems whose relevance is now well established in many different disciplines ranging from biology to linguistics and social sciences. In the simplest case of phylogeny reconstruction, one seeks the tree-like structure that better describes the evolutionary relationship between the given set of taxa. We will introduce the general concept of phylogenetic reconstruction, with a focus on distance-based reconstruction methods, i.e., methods whose starting point is a distance matrix between taxa. In this framework, we will introduce a new reconstruction scheme and we finally give an overview of interesting applications of phylogenetic reconstruction.

CARLO CARMINATI (Università di Pisa)

A canonical thickening of the rationals and the dynamics of continued fractions

We construct a countable family of open intervals contained in $(0, 1]$ whose endpoints are quadratic surds and such that their union is a full measure set. We then show that these intervals are precisely the monotonicity intervals of the entropy of α-continued fractions, thus proving a conjecture of Nakada and Natsui.

FRANCESCO CELLAROSI (Princeton University)

On the limit curlicue process for theta sums

I shall describe a random process achieved as the limit for the ensemble of curves obtained by interpolating the values of theta sums. The existence and the properties of this process are shown by means of purely dynamical tools and rely on a generalisation of results by Marklof and Jurkat and van Horne. (Joint work with J. Marklof)

NIKOLAI CHERNOV (University of Alabama at Birmingham)

Hyperbolic dynamics in 2D periodic Lorentz gases under external fields

The periodic Lorentz gas is a mathematical model of the motion of electrons in metals: a point (electron) bounces off a periodic array of fixed round obstacles (molecules). Hyperbolicity, ergodicity, and basic statistical properties (exponential decay of correlations) have been established for this model. Our main goal is to derive global ("macroscopic") features of the dynamics. When the horizon is finite (i.e., when the free path between collisions is bounded), then the particle exhibits a regular diffusion and, under a small external field and Gaussian thermostat, a regular drift (current) satisfying classical Ohm's law and the Einstein relation. Without thermostats, the particle accelerates indefinitely, but rather strangely the average drift vanishes and the motion becomes recurrent. If the horizon is infinite, then without external fields the particle's motion is characterized by an abnormal diffusion ("superdiffusion"), and with an external field and Gaussian thermostat by an abnormal current ("superconductivity"). We also introduce an alternative thermostatting mechanism (acting at the walls) that may be physically more realistic than Gaussian thermostat, and that leads us to an interesting mathematical phenomenon, non-invertible hyperbolicity.
(Joint work with D. Dolgopyat)

Stephen Coombes (University of Nottingham)
Mathematical Neuroscience: from neurons to networks

The tools of dynamical systems theory are having an increasing impact on our understanding of patterns of neural activity. In the first part of my talk I will introduce some of the more popular single neuron models and explain their behaviour in terms of bifurcation diagrams, phase-planes and phase-response curves. For limit cycle oscillators I will review the coupled oscillator approach that has provided a framework for understanding behaviour in neural networks with weak synaptic and gap junction coupling. I will then show how results for strong coupling can be obtained by focusing on a specific class of spiking neural models, namely (non-smooth) planar integrate-and-fire models. In the second part of my talk I will describe how to build tractable tissue level models that maintain a strong link with biophysical reality. These models typically take the form of nonlinear integro-differential equations. Their non-local nature has led to the development of a set of analytical and numerical tools for the study of waves, bumps and patterns, based around natural extensions of those used for local differential equation models. Here I will present an overview of these techniques, and discuss the relevance of neural field models for describing the brain at the large scales necessary for interpreting EEG data.

Jacopo De Simoi (Université Pierre et Marie Curie, Paris)
High order critical sets for some anti-integrable limits of the standard map

In this talk we present results about a family of maps which can be viewed as anti-integrable limits of the standard map. We will show how to obtain some form of equidistribution for a given class of observables by means of standard pairs and critical sets. In principle such equidistribution bounds should improve by grouping together several iterations of the map and defining the so-called high order critical sets. The talk will focus on presenting some recent results which have been obtained using the an order two critical set; we will also show some of the difficulties that arise when attempting to define higher-order critical sets and how these challenges are related to the presence of elliptic islands.

Rafael De la Llave (University of Texas at Austin)
Hyperbolic sets and their invariant manifolds in coupled map lattices: regularity and localization properties

We study coupled map lattices. We develop a formalism that allows to study perturbation theory rather comfortably. The main tool is to identify a class of functions (which we call *decay functions*) that formulate precisely that the dependence on far objects is small and which behave well under composition and other natural operations. Even if not all the arguments in finite dimensions can be adapted, some carefully chosen arguments familiar from finite dimensions can be adapted to the study of infinite systems. (Note that these systems have uncountably many periodic orbits so one cannot maintain at the same time compactness and uniform hyperbolicity).
We apply this methodology to the study of hyperbolic sets and their invariant manifolds, and we conclude persistence of the hyperbolic as well as regularity properties of the invariant manifolds, including precise statements of the decay properties of the invariant manifolds. Other applications of the framework are also possible.
(Joint work with E. Fontich and P. Martin)

Mark Demers (Fairfield University)
Escape rates, entropy and Lyapunov exponents in dynamical systems with holes

For a broad class of systems with holes, we study the relation between escape rates with respect to a reference measure and the pressure (entropy minus positive Lyapunov exponents) on the survivor set, the set of points which never enters the hole. Under certain conditions, we derive a full variational principle for the open system. When applied to Anosov diffeomorphisms, this variational principle allows us to determine how the escape rate changes as we vary the size and position of the hole.
(Joint work with P. Wright and L.-S. Young)

Michael Ghil (Ecole Normale Supérieure, Paris & University of California, Los Angeles)
Toward a mathematical theory of climate sensitivity

The first attempt at a consensus estimate of the equilibrium sensitivity of climate to changes in atmospheric CO_2 concentrations was made in the U.S. National Research Council (NRC) report of J. G. Charney and associates, in 1979. The result was the now famous range for an increase of 1.5–4.5 K in global temperatures, given a doubling of CO_2 concentration. Earth's climate, however, never was and is unlikely to ever be in equilibrium. The Intergovernmental Panel on Climate Change (IPCC) focused therefore, in addition to estimates of equilibrium sensitivity, on estimates of climate change over the 21st century. The latter estimates of temperature increase over the coming 100 years still range over several degrees Celsius. This difficulty in narrowing the range of estimates is clearly connected to the complexity of the climate system, the nonlinearity of the processes involved, and the obstacles to a faithful representation of these processes and feedbacks in IPCC-class general circulation models.
In this talk I will outline the objectives, proposed methods and preliminary results of a joint research program with M. D. Chekroun and D. Kondrashov (UCLA), E. Simonnet (INLN, Nice), and I. Zaliapin (UNR). The main objective of this program is to understand and explain, at a fundamental level, the causes and manifestations of climate sensitivity. Our program is based on weaving together recent results from three mathematical disciplines: the ergodic theory of dynamical systems, stochastic processes, and the linear response theory of nonequilibrium dynamical systems. The program's cornerstone is the theory of random dynamical systems (RDS), which allows us to probe the detailed geometric structure of the random attractors associated with nonlinear, stochastically perturbed systems. These attractors extend the concept of strange attractors from autonomous dynamical systems to nonautonomous and stochastic systems.
To illustrate our results so far, I'll show a high-resolution numerical study of two "toy" models: we obtain a good approximation of their global random attractors, as well as of the time-dependent invariant measures supported by these attractors. The latter measures are shown to be random Sinai-Ruelle-Bowen (SRB) measures; such measures have an intuitive, physical interpretation, obtained essentially by "flowing" the entire phase space onto the attractor. The first of the two models studied herein is a stochastically forced version of the classical Lorenz (1963) model. The second one is a low-dimensional, nonlinear stochastic model of the El Niño-Southern Oscillation (ENSO). While highly idealized, both these "toy" models are of fundamental interest for climate dynamics and provide insight into its predictability. Additional results will be shown for the pullback attractor of a delay-differential equation (DDE) model of ENSO, subject to periodic forcing by the seasonal cycle. If time permits, very competitive ENSO predictions based on actual climate observations and the ideas arising from the theoretical results above will also be presented.
The talk concludes with an outlook on linear response theory, including both the response function of a chaotic system to time-dependent forcing $R(t)$ and its Fourier transform, the susceptibility function $\hat{R}(\xi)$. It is especially the latter that will allow us to get a handle on mechanisms of high sensitivity in climate response to both deterministic, anthropogenic and random, natural forcing.

George Haller (McGill University, Montreal)
Lagrangian Coherent Structures, finite-time hyperbolicity and Lyapunov exponents

We review the fundamental physical motivation behind the definition of Lagrangian Coherent Structures (LCS) and show how it leads to the concept of finite-time hyperbolicity in non-autonomous dynamical systems. Using this concept of hyperbolicity, we define LCS in mathematical terms and obtain sufficient and necessary conditions for their existence in terms of finite-time Lyapunov exponents. We show several applications of LCS detection in laboratory flow experiments, as well as in atmospheric and oceanic data sets.

Stefano Isola (Università di Camerino)
Subdiffusive behaviour generated by some non-hyperbolic system

We present some results on the asymptotic behaviour of ergodic sums for simple non-hyperbolic systems, such as irrational rotations of the circle. For suitable observables these results yield information on the kind of diffusive behaviour for dynamically generated random walks.

Carlangelo Liverani (Università di Roma Tor Vergata)
Some models related to the derivation of the Fourier Law

I will describe some models, consisting of interacting mechanical systems, that are a caricature of an insulator and are amenable to rigorous investigation. To goal is to show that in the limit in which the various systems have a very weak interaction, and the time is properly rescaled, the energy exchange satisfies a stochastic equation with a structure fairly independent on the model. This is a first step in the long road of deriving the heat equation form a mechanical model.

Robert MacKay (University of Warwick)
Some examples of hyperbolic dynamics

I will

1. recall the construction of a mechanical linkage whose free dynamics is Anosov on each positive energy level, and pose problems about how to extend this to higher dimensions, and to partially hyperbolic situations, and relevance to robotics,

2. sketch the construction of a C^3-structurally stable mixing volume-preserving vector field in a bounded 3D domain with no-slip boundaries, which are essentially Anosov, and some Lorenz-like variants,

3. mention the Cerbelli-Giona map, a piecewise affine pseudo-Anosov map of the 2-torus, which allows one to construct a standard-like map with provable diffusion coefficient $1/(4\sqrt{3})$ and suggests a way to make a codimension-3 submanifold of smooth mixing standard-like maps,

4. remark that timelike geodesic flow in de Sitter space is Anosov and suggest some consequences for reinterpretation of Hubble's law.

Roberto Markarian (Universidad de la República, Montevideo)
Attractors in billiards with dominated splitting

This talk is a continuation of the one delivered in Vienna two years ago. We study billiard-type

systems whose reflections are not elastic. The existence of dominated splitting is proved for different cases.
We will present some numerical simulations and new exact results based on the method introduced in: Markarian, Pujals, Sambarino, *"Billiards with dominated splitting"*, Ergod. Th. & Dynam. Sys.
(Joint work with A. Arroyo, D. Sanders, S. Pinto and S. Oliffson)

JENS MARKLOF (University of Bristol)
Kinetic transport in the periodic Lorentz gas

The periodic Lorentz gas is one of the oldest models to study chaotic transport phenomena. In this talk I will discuss some recent results of joint work with A. Strombergsson (Uppsala), where we establish the existence of a random flight process in the Boltzmann-Grad limit (the limit of small scatterer size), and discuss the associated kinetic transport equations.

MARIA JOSÉ PACIFICO (Universidade Federal do Rio de Janeiro)
On the dynamics of the Yoccoz-Birkeland model for the Microtus Epiroticus

We study the discretized version of the dynamical system given by

$$N(t) = \int_{A_0}^{A_1} N(t-a)\, m(N(t-a))\, m_\rho(t-a)\, S(a)\, da$$

where the initial condition $N(t), t \in [-A_1, 0]$, is a continuous function and $m(N)$, $m_\rho(t)$, $S(a)$ are parameter functions that determine the evolution law. The system has been proposed by Yoccoz and Birkeland to describe the evolution of the population of *Microtus Epiroticus* (sibling vole) on Svalbard Isles. We prove that, for certain values of the parameters, the system is topologically mixing, which explains some of the high oscillations observed in Nature.

PIETRO PETERLONGO (Scuola Normale Superiore, Pisa & Ecole Normale Supérieure, Paris)
How logarithm laws may fail in a mixing system: an example with a reparametrization of a translation on the torus

Logarithm laws are ubiquitous in the ergodic theory of dynamical systems. Our attention will be focused on logarithm laws for the hitting time. In this context a tight relation can be established between logarithm law behavior and rate of decay of correlations. We will show an example of a translation on the torus for which the logarithm law, in a sense, fails. The construction of this example, highly non-generic, is based on specific properties of the translation vector which are best understood in terms of continued-fraction expansions. A suitable time-reparametrization of the suspension flow on the 3-torus allows us to obtain a mixing system which retains the same kind of pathological hitting-time behavior.
(Joint work with S. Galatolo)

LUC REY-BELLET (University of Massachusetts at Amherst)
Game theory and statistical mechanics

We discuss some connections, at the static and dynamical level, between game theory and statistical mechanics. In particular we derive and study mesoscopic equations for evolutionary game theory as well as using ideas of non-equilibrium statistical mechanics in game theory.

Domokos Szász (Budapest University of Technology)
(Super)diffusive asymtotics for perturbed Lorentz or Lorentz-like processes

After the first success in establishing the diffusive, Brownian limit of planar, finite-horizon, periodic Lorentz processes, in 1981 Sinai turned the interest toward studying models when periodicity is hurt, in particular, to locally perturbed Lorentz processes. The 1981 solution for a stochastic random-walk-model only led in 2009 to that for the locally perturbed, finite-horizon Lorentz process (by Dolgopyat, Varjú and the present author). Beside reporting on these results we also analyze the first steps in extending the super-diffusive limit obtained for the infinite horizon Lorentz process to locally perturbed ones (results by Nándori, Paulin, Varjú and the speaker).

Péter Tóth (Budapest University of Technology and Economics)
A heat conduction model with localized billiard disks and weak interaction forces

I study a heat conduction model with localized billiard disks interacting via conservative forces. This is a natural modification of the popular Gaspard-Gilbert model. In the weak coupling limit (with time rescaled appropriately) the energies at the lattice sites form a Markov chain, actually a set of coupled diffusion processes with continuous trajectories (unlike in the Gaspard-Gilbert model).
The hydrodynamic-limit behavior of this system is understood through a mixture of rigorous, heuristic and numerical methods. The heat conductivity turns out to decrease with temperature as $T^{-3/2}$, which is in surprisingly good agreement with certain experimental data.

Masato Tsujii (Kyushu University, Fukuoka)
Contact Anosov flows and FBI transform

In this talk, I would like to speak about the relation between the study of transfer operators for geodesic flows on negatively curved manifolds and semi-classical analysis. In studying transfer operators for hyperbolic flows, the main difficulty (and interest) lies in the analysis of their action on functions having high frequency in the flow direction. This leads us to the situation where functions with high frequency are push-forwarded by the geodesic flow and reminds us of the main theme of semi-classical analysis. So naturally we expect that some ideas from semi-classical analysis may be useful in the study of transfer operators for "classical" geodesic flows. Here we try to use a modification of the FBI (Fourier-Bros-Iaglonitzer) transform, which is a kind of wave-packet decomposition, to study transfer operators for geodesic flows on negatively curved manifolds. As a result, we show quasi-compactness of those transfer operators. This result is actually the same as I announced a few years ago. But now the proof is much more transparent.

Corinna Ulcigrai (University of Bristol)
Mixing time-changes of parabolic flows

In this talk we will focus on some of the ergodic properties of parabolic flows and their time reparametrizations. We will mention mixing properties of flows on surfaces and then concentrate on another fundamental example of parabolic dynamical systems, Heisenberg nilflows. The latter are flows on quotients of the Heisenberg group and it is well known that they are not mixing. We show that there is a set of time changes so that the corresponding reparametrizations are mixing. This set is generic in a appropriate sense and can be explicitely described.
(Joint work with A. Avila and G. Forni)

Tanya Yarmola (University of Maryland)
Ergodicity of some open systems with particle-disk interactions

We consider a bounded domain on the plane containing N rotating disks pinned down at their centers. The system is coupled to heat baths that absorb and emit particles through several openings on the boundary of the domain. The particles do not interact with each other, have specular collisions with the boundary of the domain, and exchange energy with the disks.
For certain classes of such systems, we show that if there exists an invariant measure with support away from the states with "trapped trajectories", then it is both absolutely continuous and ergodic. The key properties of the system that lead to absolute continuity and ergodicity are randomness of the injection process and ability to control angular velocities of the disks through appropriate particle injections.

James Yorke (University of Maryland)
Infinitely many cascades must exist as chaos arises in $\mathbb{R}^n$

Evelyn Sander and I have established a general theory of why period-doubling cascades exist (in n dimensions), including why smooth systems have infinitely many cascades. Feigenbaum's results describe how a cascade's bifurcations scale—if the cascade exists. Collet, Eckmann, and Lanford extended these results by 1980 to argue that almost every cascade of sufficiently smooth maps have Feigenbaum-like scaling. They did not—except in special cases—show that cascades exist. We fill that gap using topological arguments.
We show that infinitely many cascades must exist as a system goes from having only finitely many periodic orbits to having chaotic dynamics. Our theory is for generic smooth one-parameter maps $F(\mu, x)$ where x is n-dimensional. Here is one corollary for maps with horseshoes in dimension 2 such as the time-1 map of the forced damped pendulum or double-well Duffing equation.

The Route to Chaos Theorem Assume $F(\mu, x)$ is smooth and x is two-dimensional. Under additional mild restrictions, if there are parameter values μ_0 and μ_1 for which $F(\mu_0, \cdot)$ has at most a finite number of periodic orbits, and $F(\mu_1, \cdot)$ has exponential growth of number of periodic orbits as a function of the period. Then there are infinitely many period-doubling cascades between μ_0 and μ_1.

The above result also holds when x is one dimensional with minor wording changes. In addition we have discovered a new phenomenon in which there are paired cascades, that is, two cascades that are connected by a path of periodic orbits. The quadratic map $\mu - x^2$ has no paired cascades but almost all cascades are paired for the forced damped pendulum and for the forced single and double-well Duffing equations.

Lai-Sang Young (New York University)
Dynamics of periodically-kicked oscillators

Under suitable conditions, the impact of a kick can be substantially magnified by the underlying shear in a limit cycle, leading to the formation of horseshoes and strange attractors. I will discuss this mechanism in some detail using a simple linear shear model. An important point I would like to convey is that the phenomenon described manifests itself (in slightly different form) in many different contexts. I will mention briefly two examples: white-noise forcing and reaction-diffusion equations undergoing Hopf bifurcations.