INDAM MEETING:

HYPERBOLIC DYNAMICAL SYSTEMS IN THE SCIENCES

CORINALDO (ITALY)
MAY 31 - JUNE 4, 2010

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Hyperbolic dynamics in 2D periodic Lorentz gases under external fields

The periodic Lorentz gas is a mathematical model of the motion of electrons in metals: a point (electron) bounces off a periodic array of fixed round obstacles (molecules). Hyperbolicity, ergodicity, and basic statistical properties (exponential decay of correlations) have been established for this model. Our main goal is to derive global ("macroscopic") features of the dynamics. When the horizon is finite (i.e., when the free path between collisions is bounded), then the particle exhibits a regular diffusion and, under a small external field and Gaussian thermostat, a regular drift (current) satisfying classical Ohm's law and the Einstein relation. Without thermostats, the particle accelerates indefinitely, but rather strangely the average drift vanishes and the motion becomes recurrent. If the horizon is infinite, then without external fields the particle's motion is characterized by an abnormal diffusion ("superdiffusion"), and with an external field and Gaussian thermostat by an abnormal current ("superconductivity"). We also introduce an alternative thermostatting mechanism (acting at the walls) that may be physically more realistic than Gaussian thermostat, and that leads us to an interesting mathematical phenomenon, non-invertible hyperbolicity.
(Joint work with D. Dolgopyat)