INDAM MEETING:

HYPERBOLIC DYNAMICAL SYSTEMS IN THE SCIENCES

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Toward a mathematical theory of climate sensitivity

The first attempt at a consensus estimate of the equilibrium sensitivity of climate to changes in atmospheric CO_2 concentrations was made in the U.S. National Research Council (NRC) report of J. G. Charney and associates, in 1979. The result was the now famous range for an increase of 1.5–4.5 K in global temperatures, given a doubling of CO_2 concentration. Earth's climate, however, never was and is unlikely to ever be in equilibrium. The Intergovernmental Panel on Climate Change (IPCC) focused therefore, in addition to estimates of equilibrium sensitivity, on estimates of climate change over the 21st century. The latter estimates of temperature increase over the coming 100 years still range over several degrees Celsius. This difficulty in narrowing the range of estimates is clearly connected to the complexity of the climate system, the nonlinearity of the processes involved, and the obstacles to a faithful representation of these processes and feedbacks in IPCC-class general circulation models.
In this talk I will outline the objectives, proposed methods and preliminary results of a joint research program with M. D. Chekroun and D. Kondrashov (UCLA), E. Simonnet (INLN, Nice), and I. Zaliapin (UNR). The main objective of this program is to understand and explain, at a fundamental level, the causes and manifestations of climate sensitivity. Our program is based on weaving together recent results from three mathematical disciplines: the ergodic theory of dynamical systems, stochastic processes, and the linear response theory of nonequilibrium dynamical systems. The program's cornerstone is the theory of random dynamical systems (RDS), which allows us to probe the detailed geometric structure of the random attractors associated with nonlinear, stochastically perturbed systems. These attractors extend the concept of strange attractors from autonomous dynamical systems to nonautonomous and stochastic systems.
To illustrate our results so far, I'll show a high-resolution numerical study of two "toy" models: we obtain a good approximation of their global random attractors, as well as of the time-dependent invariant measures supported by these attractors. The latter measures are shown to be random Sinai-Ruelle-Bowen (SRB) measures; such measures have an intuitive, physical interpretation, obtained essentially by "flowing" the entire phase space onto

the attractor. The first of the two models studied herein is a stochastically forced version of the classical Lorenz (1963) model. The second one is a low-dimensional, nonlinear stochastic model of the El Niño-Southern Oscillation (ENSO). While highly idealized, both these “toy” models are of fundamental interest for climate dynamics and provide insight into its predictability. Additional results will be shown for the pullback attractor of a delay-differential equation (DDE) model of ENSO, subject to periodic forcing by the seasonal cycle. If time permits, very competitive ENSO predictions based on actual climate observations and the ideas arising from the theoretical results above will also be presented.

The talk concludes with an outlook on linear response theory, including both the response function of a chaotic system to time-dependent forcing $R(t)$ and its Fourier transform, the susceptibility function $\hat{R}(\xi)$. It is especially the latter that will allow us to get a handle on mechanisms of high sensitivity in climate response to both deterministic, anthropogenic and random, natural forcing.