Toward a Mathematical Theory of Climate Sensitivity

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Please visit these sites for more info.

http://www.atmos.ucla.edu/tcd/
http://www.environnement.ens.fr/

Motivation

- The ***climate system*** is highly ***nonlinear and*** quite ***complex***.
- Its ***major components*** – the atmosphere, oceans, ice sheets – ***evolve on many*** space and time ***scales***.
- Its ***predictive understanding*** has to rely on the system's physical, chemical and biological ***modeling,***

  but also on the ***mathematical analysis*** of the models

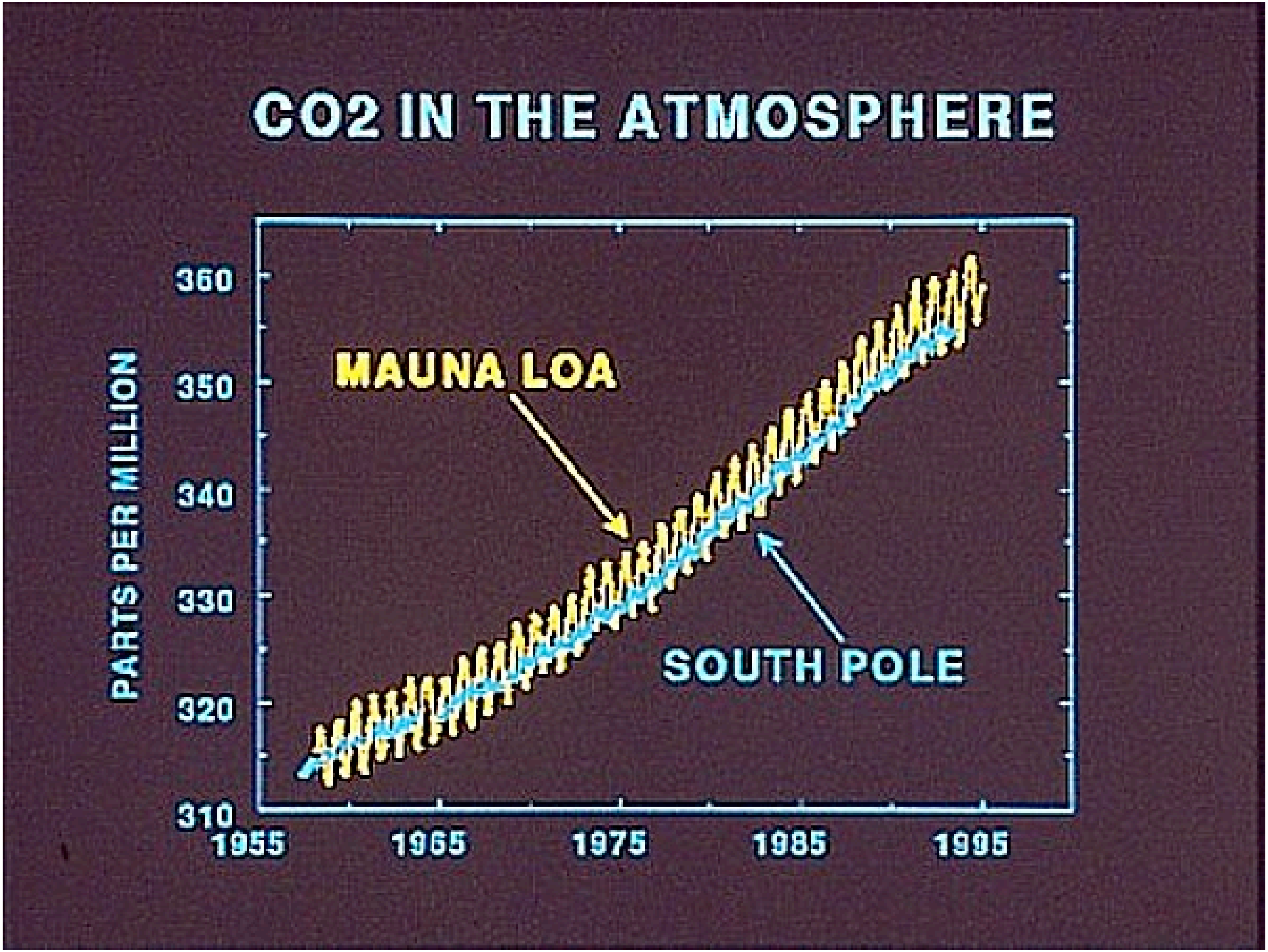
  thus obtained.
- The ***hierarchical modeling*** approach allows one to

  give proper weight to the ***understanding*** provided by the

  models ***vs.*** their ***realism***, respectively.
- Back-and-forth between ***"toy"*** (conceptual) and ***detailed*** ("realistic") ***models***, and between ***models*** and ***data***.
- Such an approach facilitates the evaluation of ***forecasts (prognostications?)*** based on these models.

Outline

- The IPCC process: results and further questions
- Natural climate variability as a source of uncertainties
  - sensitivity to initial data ➔ error growth
  - sensitivity to model formulation ➔ see below!
- Uncertainties and how to fix them
  - structural in/stability
  - random dynamical systems (RDS)
- Two or more illustrative examples
  - Arnol'd tongues and a "French garden"
  - the Lorenz system
  - an ENSO "toy" model
- Linear response theory and climate sensitivity
- Conclusions, work in progress and references


CO2 IN THE ATMOSPHERE
PARTS PER MILLION
360
350
340
330
320
310
1955
1965
1975
1985
1995
MAUNA LOA
SOUTH POLE

Unfortunately, things aren't all that easy!

What to do?

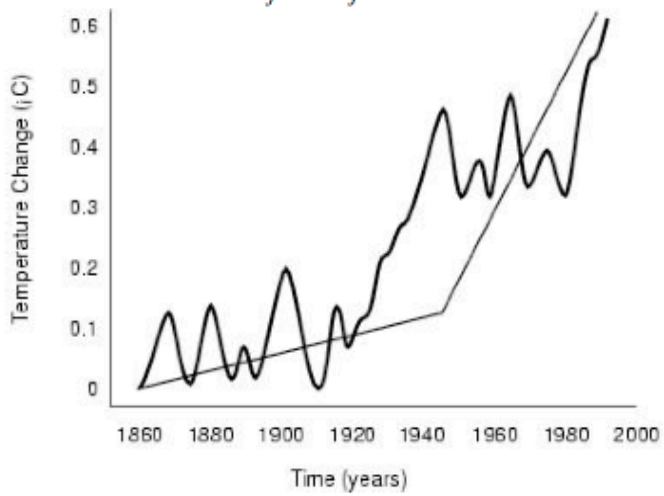
Try to achieve better interpretation of, and agreement between, models …

Ghil, M., 2002: Natural climate variability, in *Encyclopedia of Global Environmental Change*, T. Munn (Ed.), Vol. 1, Wiley

Natural variability introduces additional complexity into the anthropogenic climate change problem

The most common interpretation of observations and GCM simulations of climate change is still in terms of a scalar, linear Ordinary Differential Equation (ODE)

$$c\frac{dT}{dt} = -kT + Q \qquad \begin{array}{l} k = \sum k_i - \text{feedbacks (+ve and -ve)} \\ Q = \sum Q_j - \text{sources \& sinks} \\ Q_j = Q_j(t) \end{array}$$



Linear response to CO_2 vs. observed change in T

Hence, we need to consider instead a system of nonlinear Partial Differential Equations (PDEs), with parameters and multiplicative, as well as additive forcing (deterministic + stochastic)

$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{N}(\boldsymbol{X}, t, \mu, \beta)$$

Global warming and its socio-economic impacts

Temperatures rise:

- What about impacts?
- How to adapt?

The answer, my friend, is blowing in the wind, *i.e.*, it depends on the accuracy and reliability of the forecast …

Source : IPCC (2007), *AR4, WGI, SPM*

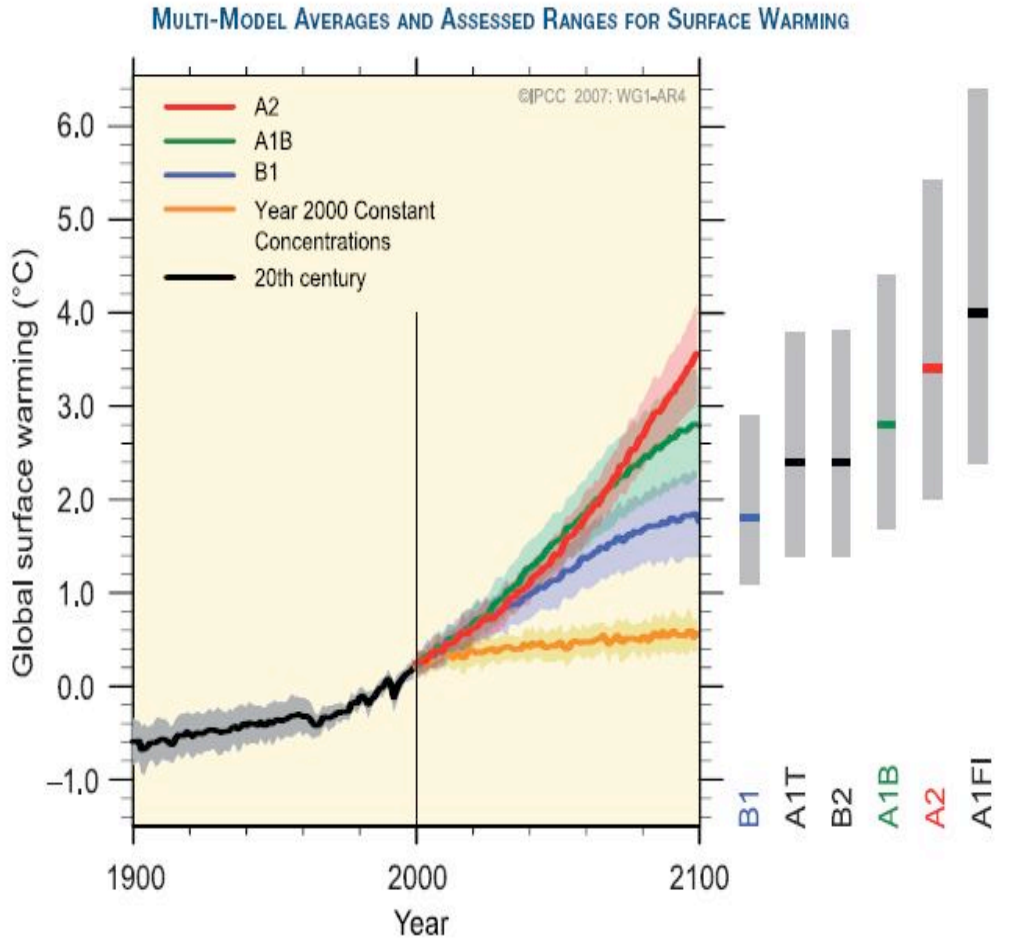
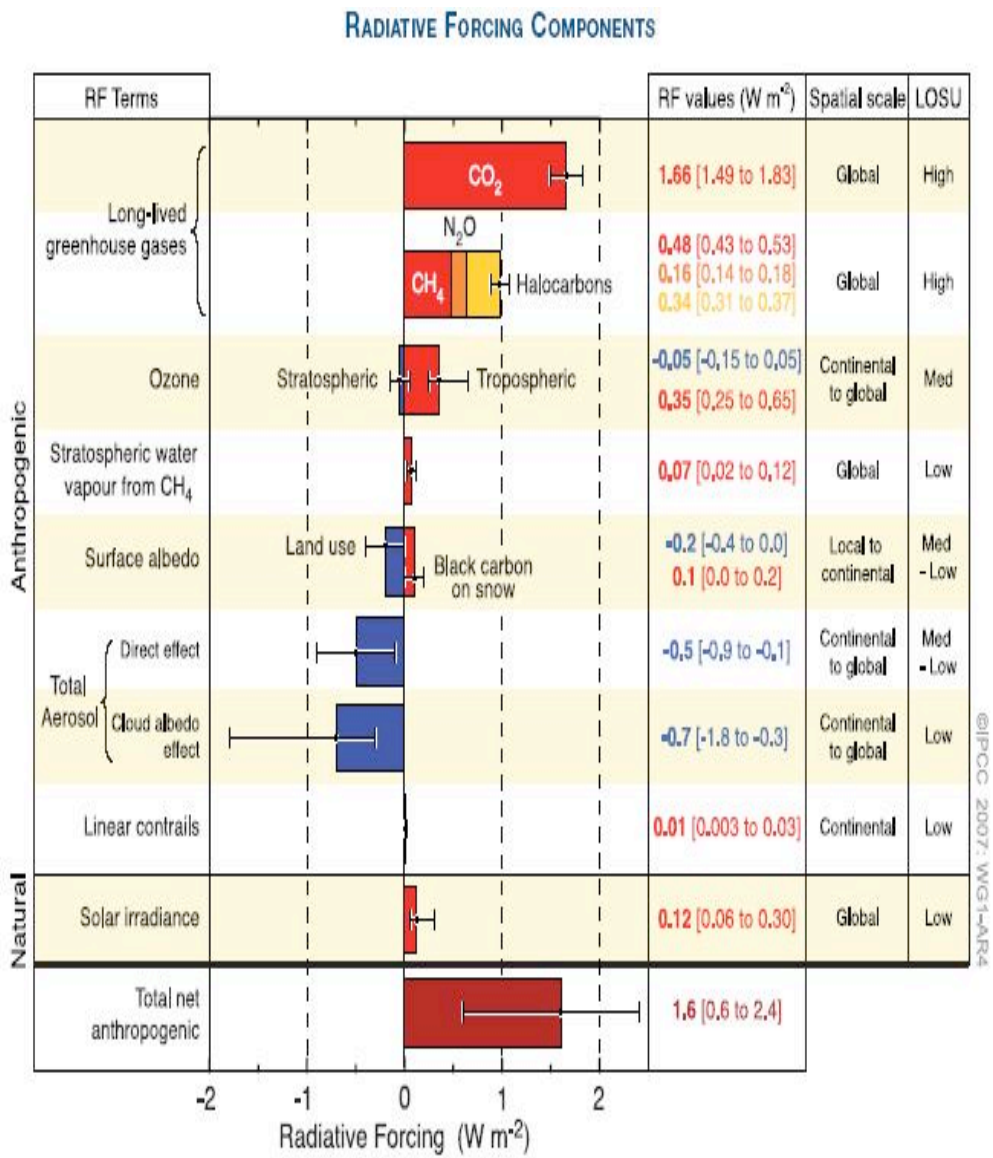


Figure SPM.5. *Solid lines are multi-model global averages of surface warming (relative to 1980–1999) for the scenarios A2, A1B and B1, shown as continuations of the 20th century simulations. Shading denotes the ±1 standard deviation range of individual model annual averages. The orange line is for the experiment where concentrations were held constant at year 2000 values. The grey bars at right indicate the best estimate (solid line within each bar) and the* ***likely*** *range assessed for the six SRES marker scenarios. The assessment of the best estimate and* ***likely*** *ranges in the grey bars includes the AOGCMs in the left part of the figure, as well as results from a hierarchy of independent models and observational constraints. {Figures 10.4 and 10.29}*

GHGs rise!

It's gotta do with us, at least a bit, ain't it?

But just how much?



IPCC (2007)

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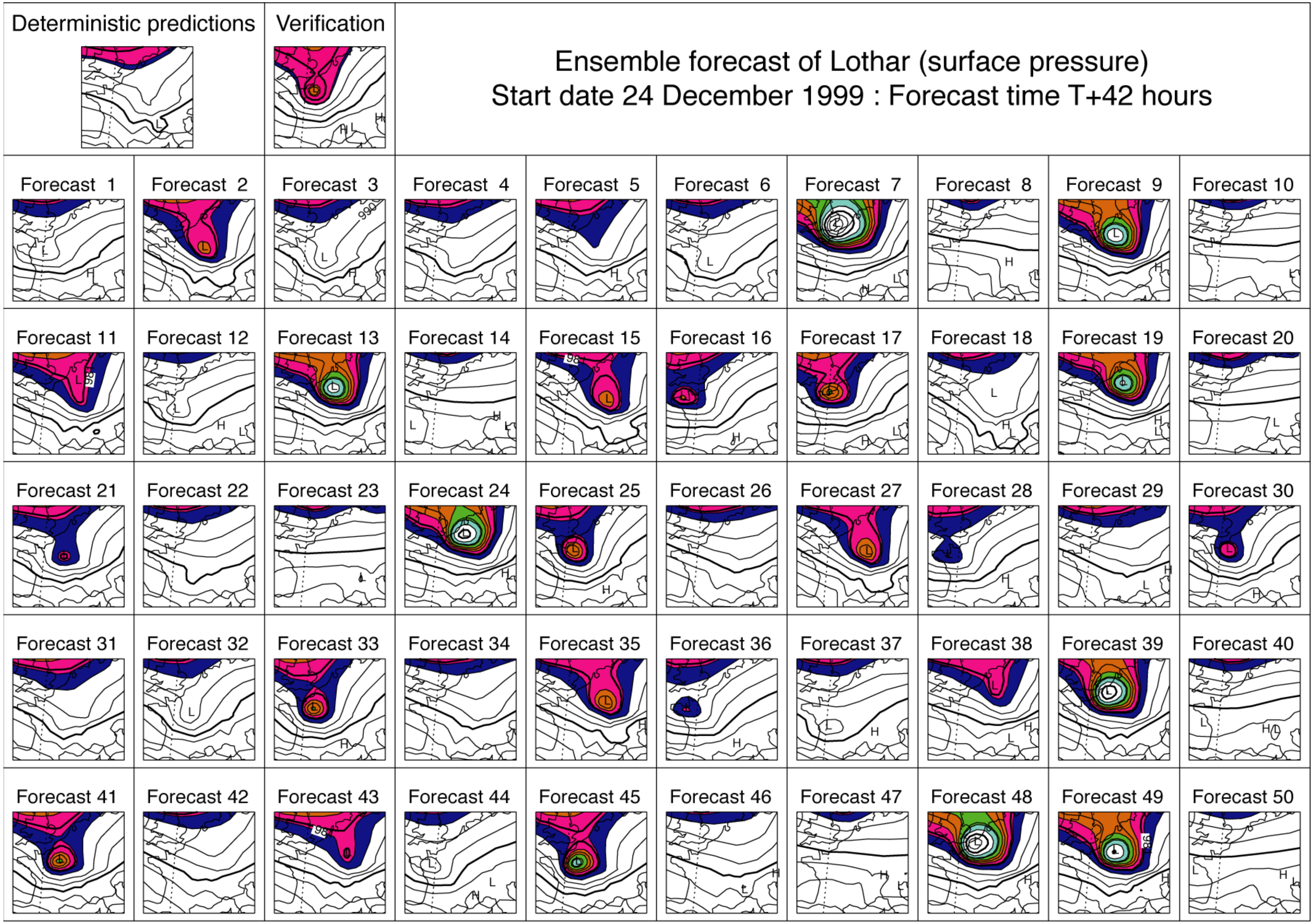

Courtesy Tim Palmer, 2009

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Can we, nonlinear dynamicists, help?

The uncertainties
might be ***intrinsic***,
rather than mere
"tuning problems"

If so, maybe
stochastic structural stability could help!

Might fit in nicely with
recent taste for
"stochastic
parameterizations"

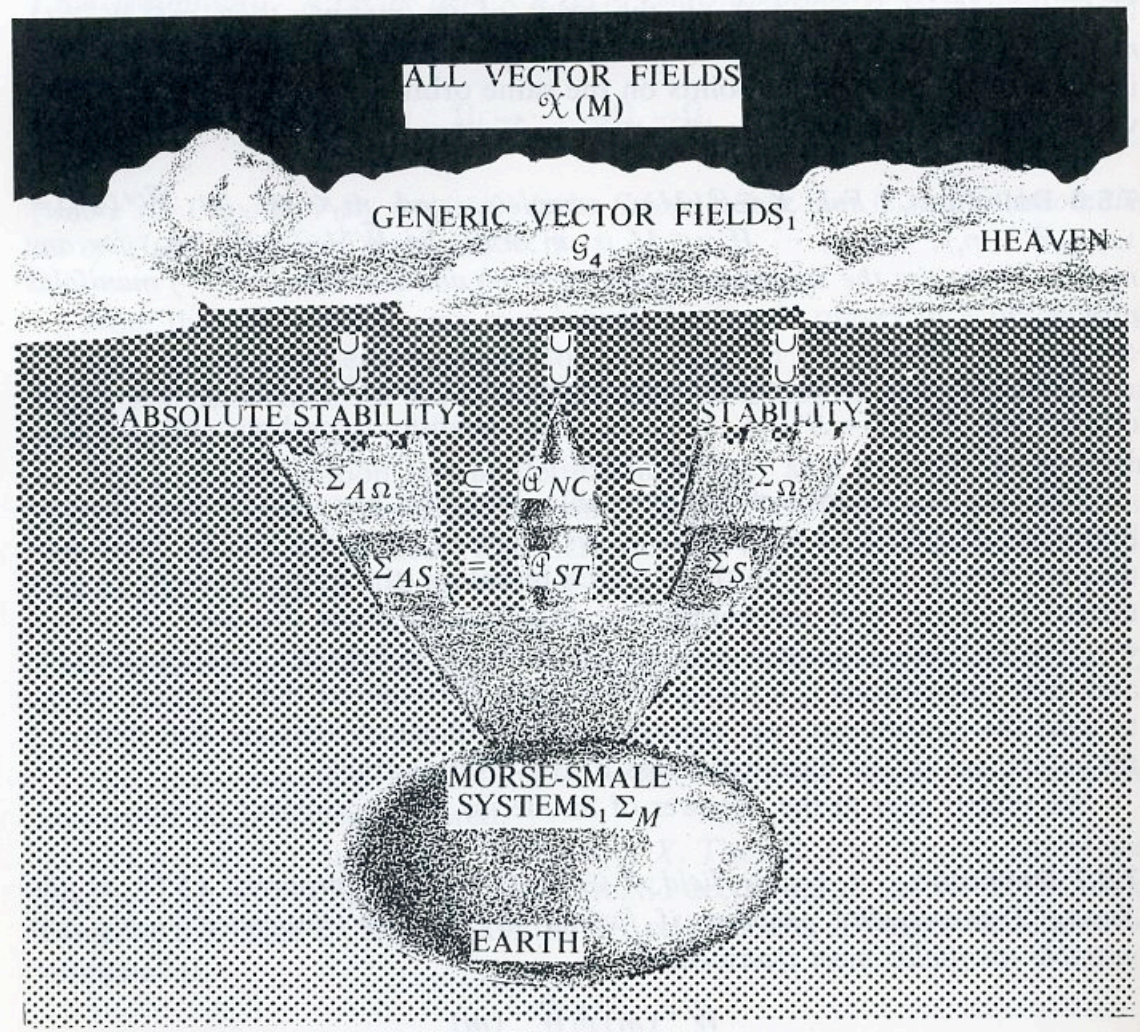

Figure 7.5-1. The three towers of differentiable dynamics.

The DDS dream of structural stability (from *Abraham & Marsden*, 1978)

So what's it gonna be like, by 2100?

Table SPM.2. *Recent trends, assessment of human influence on the trend and projections for extreme weather events for which there is an observed late-20th century trend. {Tables 3.7, 3.8, 9.4; Sections 3.8, 5.5, 9.7, 11.2–11.9}*

Phenomenon[a] and direction of trend	Likelihood that trend occurred in late 20th century (typically post 1960)	Likelihood of a human contribution to observed trend[b]	Likelihood of future trends based on projections for 21st century using SRES scenarios
Warmer and fewer cold days and nights over most land areas	*Very likely*[c]	*Likely*[d]	*Virtually certain*[d]
Warmer and more frequent hot days and nights over most land areas	*Very likely*[e]	*Likely (nights)*[d]	*Virtually certain*[d]
Warm spells/heat waves. Frequency increases over most land areas	*Likely*	*More likely than not*[f]	*Very likely*
Heavy precipitation events. Frequency (or proportion of total rainfall from heavy falls) increases over most areas	*Likely*	*More likely than not*[f]	*Very likely*
Area affected by droughts increases	*Likely* in many regions since 1970s	*More likely than not*	*Likely*
Intense tropical cyclone activity increases	*Likely* in some regions since 1970	*More likely than not*[f]	*Likely*
Increased incidence of extreme high sea level (excludes tsunamis)[g]	*Likely*	*More likely than not*[f,h]	*Likely*[i]

A linear example as a paradigm

Let us first start with a very difficult problem:

$$\text{Study the "dynamics" of} \quad \dot{x} = -\alpha x + \sigma t, \quad \alpha, \sigma > 0. \tag{1}$$

First remarks:

- The system $\dot{x} = -\alpha x$, i.e. the autonomous part of (1), is dissipative. All the solutions of $\dot{x} = -\alpha x$, converge towards 0 as $t \to +\infty$.
- Is it the case for (1)? Certainly not! The autonomous part is forced; we even introduce an infinite energy over an infinite horizon: $\int_0^{+\infty} t \, dt = +\infty$! Forward attraction seems to be ill adapted to time-dependent forcing.

Goal:

Find a concept of attraction such that:

(i) It is compatible with the forward concept, when there is no forcing,

(ii) It provides a way to assess the effect of dissipation in some sense.

For that let's do some computations...

Non-autonomous Dynamical Systems

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Non-autonomous Dynamical Systems

Commentaries

- We've just shown that:

$$|x(t,s;x_0) - a(t)| \underset{s \to -\infty}{\longrightarrow} 0 \text{ ; for every } t \text{ fixed,}$$

all initial data x_0, with $a(t) = \frac{\sigma}{\alpha}(t - 1/\alpha)$.

- We've just encountered the concept of pullback attraction; here $\{a(t)\}$ is the pullback attractor of the system (1).
- What does it means physically?
The pullback attractor provides a way to assess an asymptotic regime at time t — the time at which we observe the system — for a system starting to evolve from the remote past s, $s << t$.
- Thus, this asymptotic regime evolves with time: it is a dynamical object.
- The effect of dissipation is now viewed via this dynamical object and not a static one, as a strange attractor does for autonomous systems.

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Random Dynamical Systems - RDS theory

This theory is a combination of measure (probability) theory and dynamical systems developed by the “Bremen group" (L.Arnold, 1998). It allows one to treat Stochastic Differential Equations (**SDEs**), and more general systems driven by some “noise," as **flows**.

Setting:

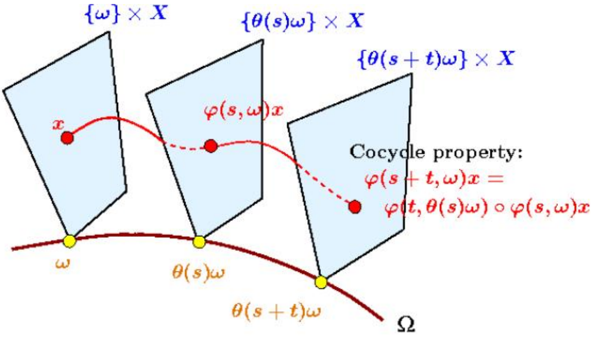
(i) A phase space X. **Example**: $\mathbb{R}^n$.

(ii) A probability space $(\Omega, \mathcal{F}, \mathbb{P})$. **Example**: The Wiener space $\Omega = C_0(\mathbb{R}; \mathbb{R}^n)$ with Wiener measure $\mathbb{P} = \gamma$.

(iii) A model of the noise $\theta(t) : \Omega \to \Omega$ that preserves the measure $\mathbb{P}$, i.e. $\theta(t)\mathbb{P} = \mathbb{P}$; θ is called the driving system. **Example**: $W(t, \theta(s)\omega) = W(t+s, \omega) - W(s, \omega)$; it starts the noise at s instead of $t = 0$.

(iv) A mapping $\varphi : \mathbb{R} \times \Omega \times X \to X$ with the cocycle property. **Example**: The solution of an SDE.

Random Dynamical Systems - A geometric view of SDEs



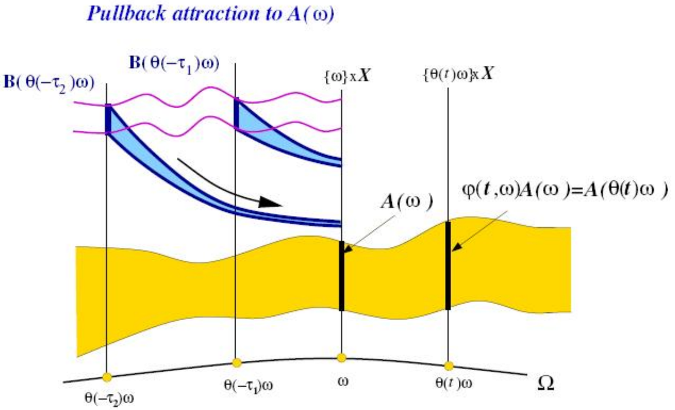
- φ **is a random dynamical system (RDS)**
- $\Theta(t)(x,\omega) = (\theta(t)\omega, \varphi(t,\omega)x)$ **is a flow on the bundle**

Random Dynamical Systems - Random attractor

A random attractor $\mathcal{A}(\omega)$ is both *invariant* and "pullback" *attracting*:

(a) **Invariant**: $\varphi(t,\omega)\mathcal{A}(\omega) = \mathcal{A}(\theta(t)\omega)$.

(b) **Attracting**: $\forall B \subset X$, $\lim_{t\to\infty} \mathrm{dist}(\varphi(t,\theta(-t)\omega)B, \mathcal{A}(\omega)) = 0$ a.s.



A tool for classification: stochastic equivalence

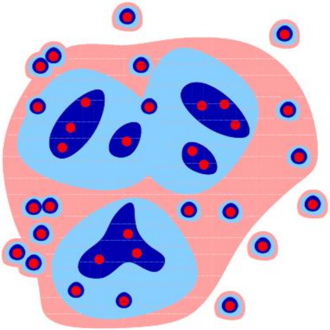
- Stochastic equivalence: two cocycles $\varphi_1(t,\omega)$ and $\varphi_2(t,\omega)$ are conjugated iff there exists a **random homeomorphism** $h \in \mathrm{Homeo}(X)$ and an invariant set $\tilde{\Omega}$ of full $\mathbb{P}$-measure (w.r.t. θ) such that $h(\omega)(0) = 0$ and:

$$\varphi_1(t,\omega) = h(\theta(t)\omega)^{-1} \circ \varphi_2(t,\omega) \circ h(\omega); \qquad (2)$$

  h is also called cohomology of φ_1 and φ_2. It is a **random change of variables**!

- Motivation: We would like to measure quantitatively as well as quantitatively the difference between climate models.

Stochastic equivalence - Could noise help the classification?



As the noise variance tends to zero and/or the parametrizations are switched off, one recovers the structural instability, as a “granularity" of model space. For nonzero variance, the random attractor $\{\mathcal{A}(\omega)\}$ associated with several GCMs might fall into larger and larger classes as the noise level increases.

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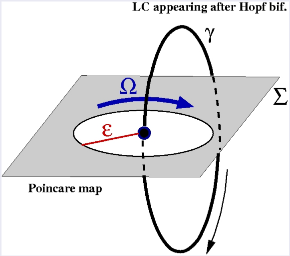
Investigation of these ideas on a family of dynamical *toy* systems - Theoretical and numerical results

V. Arnold's family of diffeomorphisms

- We want to perform a *classification* in terms of stochastic equivalence.
- Our first theoretical laboratory is **Arnold's family of diffeomorphisms of the circle:**

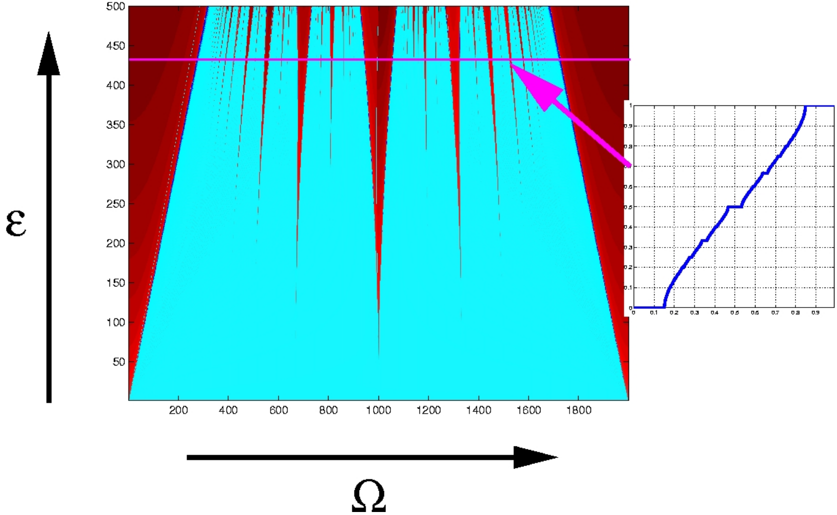
$$x_{n+1} = F_{\Omega,\varepsilon}(x_n) := x_n + \Omega - \varepsilon \sin(2\pi x_n) \quad \mod 1$$



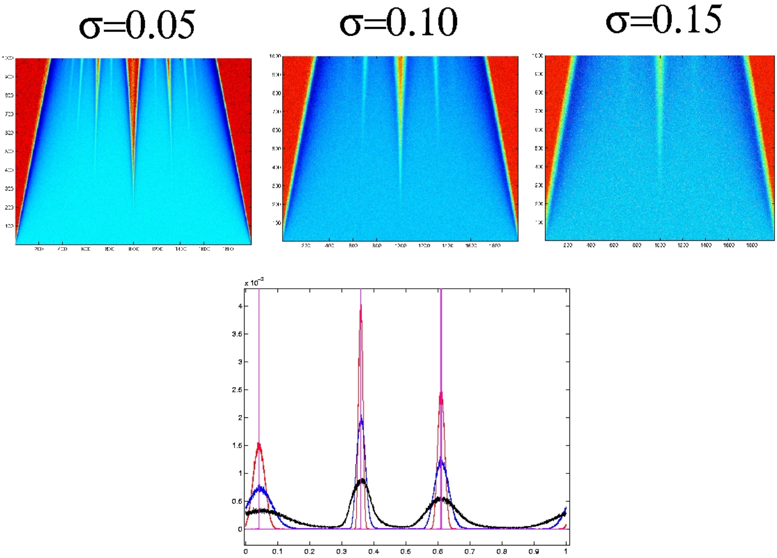
Which paradigm is represented by this family? Why this family?

- Frequency-locking phenomena & Devil's staircase
- **Topological classification** of Arnold's family $\{F_{\Omega,\varepsilon}\}$:
  - **Countable** regions of structural stability,
  - **Uncountable** structurally unstable systems with **non-zero Lebesgue measure!**
- **Two types of attractors:**
  - Periodic orbits in the circle.
  - The whole circle.

Arnold's tongues and Devil's staircase



Effect of the noise on topological classification?



Effect of the noise on the PDF of Arnold's tongue 1/3

Extension of the paradigm - Devil's quarry

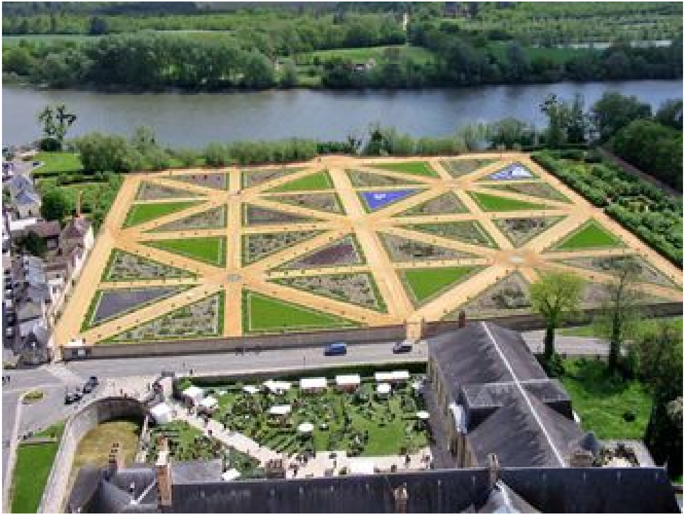
Short description of the deterministic model

- Dynamics on a 2-D torus:

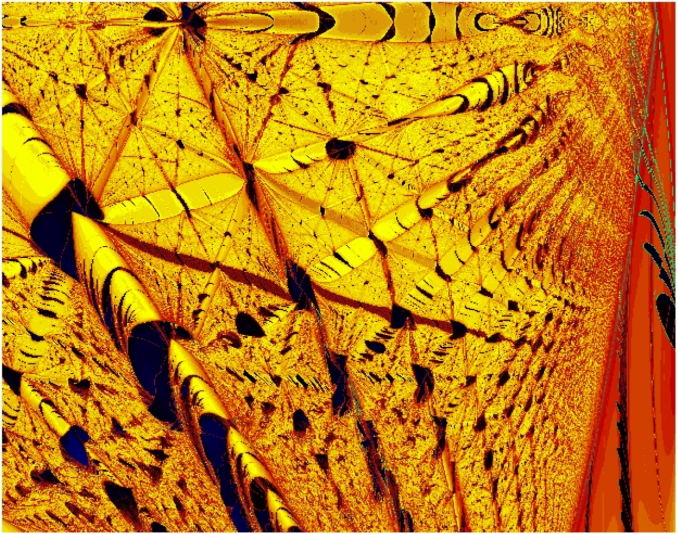
$$x_{n+1} = x_n + \Omega_1 - \varepsilon \sin(2\pi y_n), \quad \text{mod } 1$$
$$y_{n+1} = y_n + \Omega_2 - \varepsilon \sin(2\pi x_n) \quad \text{mod } 1$$

- Web of resonances & chaos:
  - Partial resonance (Ω_1, Ω_2 are rational and there is one rational relation $m_1\Omega_1 + m_2\Omega_2 = k \in \mathbb{Z}^*$ with $(m_1, m_2) \in \mathbb{Z}^* \times \mathbb{Z}^*$)
  - Full resonance
  - Chaos with possibly multiple attractors
- A more realistic paradigm of observed dynamics in the geosciences, and more...
- What is the effect of noise in such a context?

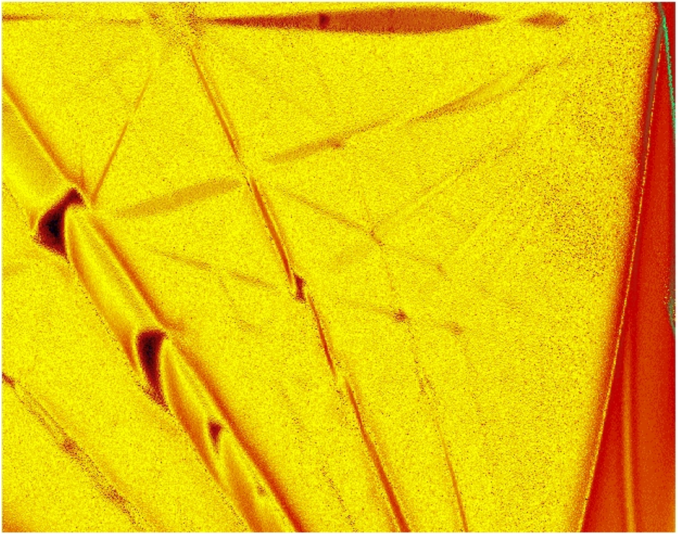
A French garden near the castle of La Roche-Guyon



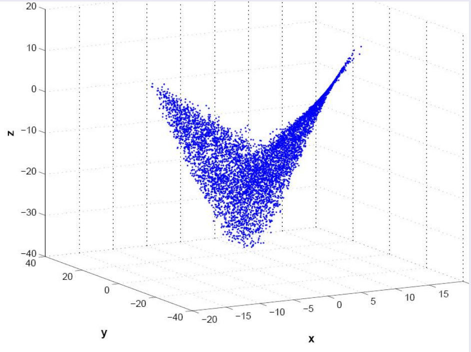
Devil's quarry for a coupling parameter $\varepsilon = 0.15$: a web of resonances



Effect of the noise on Devil's quarry

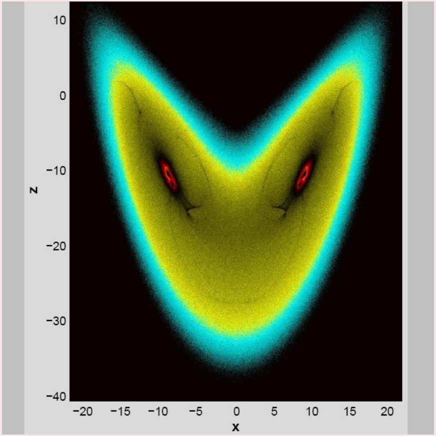


Snapshot of the random attractor (RA)



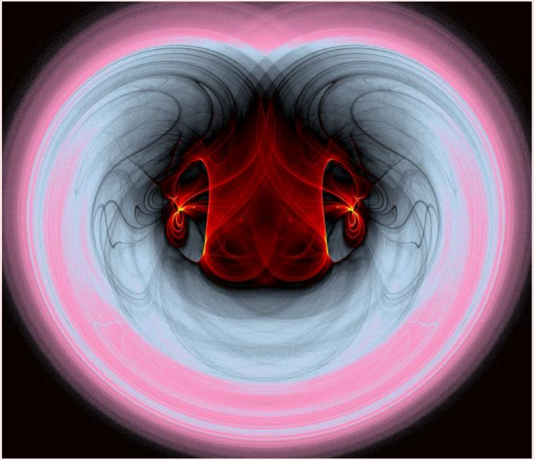
- A snapshot of the RA, $\mathcal{A}(\omega)$, computed at a fixed time t and for the same realization ω; it is made up of points transported by the stochastic flow, from the remote past $t-T$, $T >> 1$.
- We use small multiplicative noise in the deterministic Lorenz model, with the classical parameter values $b = 8/3$, $\sigma = 10$, and $r = 28$.
- Even computed pathwise, this object supports meaningful statistics.

Disintegration of the measure supported by the Lorenz R.A.



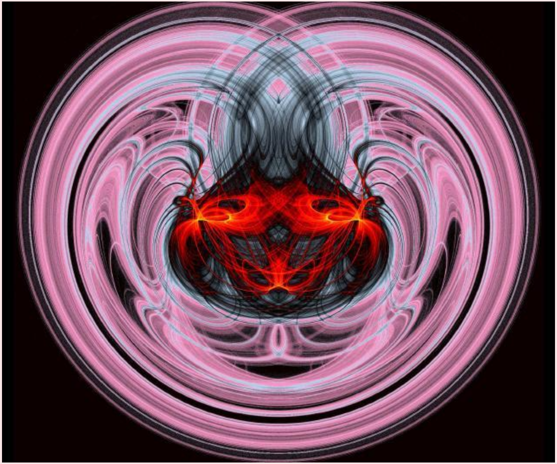
- We can compute the probability measure on the R.A. at some fixed time t. We show a "projection", $\int \mu_\omega(x, y, z)\mathrm{d}y$, with multiplicative noise: $\mathrm{d}x_i$=Lorenz$(x_1, x_2, x_3)\mathrm{d}t$ + α $x_i\mathrm{d}W_t$; $i \in \{1, 2, 3\}$.
- 10 million of initial points have been used for this picture!

Disintegration of the measure supported by the R.A.

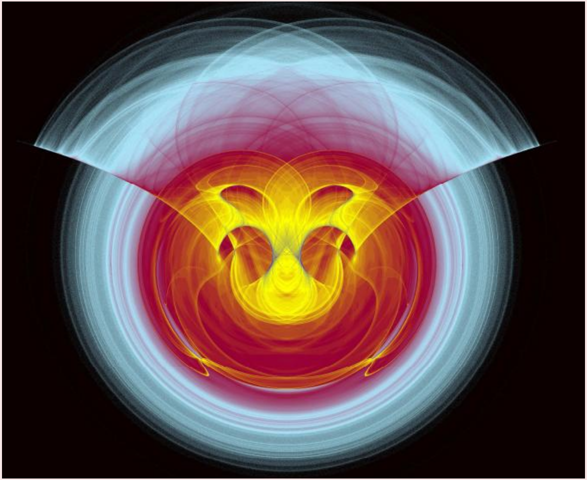


- Still 1 Billion I.D., and $\alpha = 0.3$.

Disintegration of the measure supported by the R.A.



- Still 1 Billion I.D., and $\alpha = 0.5$. Another one?



- Here $\alpha = 0.4$. The sample measure is approximated for another realization of the noise, starting from 8 billion I.D.
- Now more serious stuff is coming...

Disintegration of the measure supported by the R.A.

Disintegrations of the measure evolve with time.

- Recall that these disintegrated measures are the frozen statistics at a time t for a realization ω.
- How do these frozen statistics evolve with time?
- Action!

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Disintegration of the measure supported by the R.A.

Disintegrations of the measure evolve with time.

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- How do these frozen statistics evolve with time?
- Action!

Applications to a non-linear stochastic El Niño model

Simonnet, C. and Ghil, 2008

Timmerman & Jin (*Geophys. Res. Lett.*, 2002) have derived the following low-order, tropical-atmosphere–ocean model. The model has three variables: thermocline depth anomaly h, and
SSTs T_1 and T_2 in the western and eastern basin.

$$\begin{aligned}
\dot{T}_1 &= -\alpha(T_1 - T_r) - \frac{2\varepsilon u}{L}(T_2 - T_1),\\
\dot{T}_2 &= -\alpha(T_2 - T_r) - \frac{w}{H_m}(T_2 - T_{sub}),\\
\dot{h} &= r(-h - bL\tau/2).
\end{aligned}$$

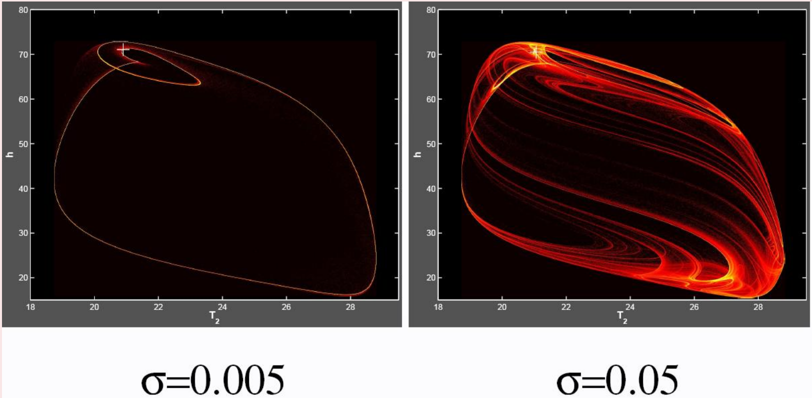
The related diagnostic equations are:

$$\begin{aligned}
T_{sub} &= T_r - \frac{T_r - T_{r0}}{2}[1 - \tanh(H + h_2 - z_0)/h^*]\\
\tau &= \frac{a}{\beta}(T_1 - T_2)[\xi_t - 1].
\end{aligned}$$

- τ: the wind stress anomalies, $w = -\beta\tau/H_m$: the equatorial upwelling.
- $u = \beta L\tau/2$: the zonal advection, T_{sub}: the subsurface temperature.

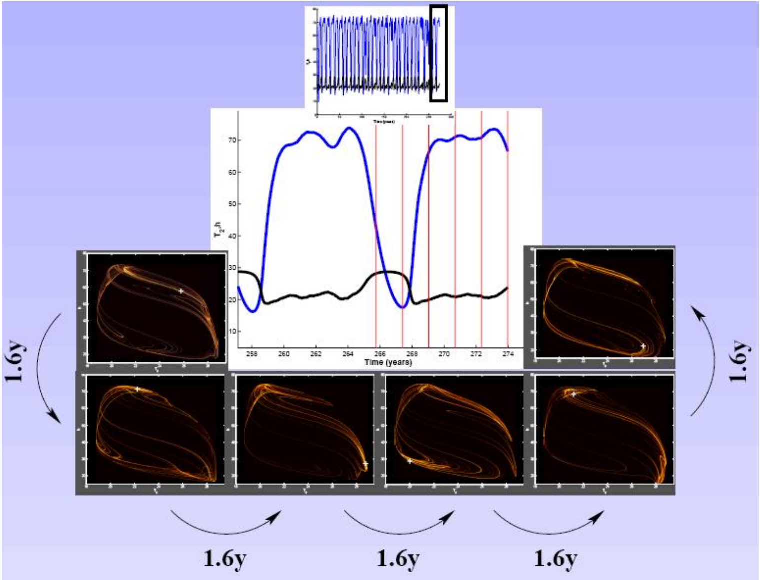
Wind stress bursts are modeled as white noise ξ_t of variance σ,
and ε measures the strength of the zonal advection.

Random Shil'nikov horseshoes



- Horseshoes can be noise-excited, left: a weakly-perturbed limit cycle, right: the same with larger noise.
- Golden: most frequently-visited areas; white 'plus' sign: most visited.

An episode in the random's attractor life



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Climate and Its Sensitivity

Let's say CO_2 doubles:

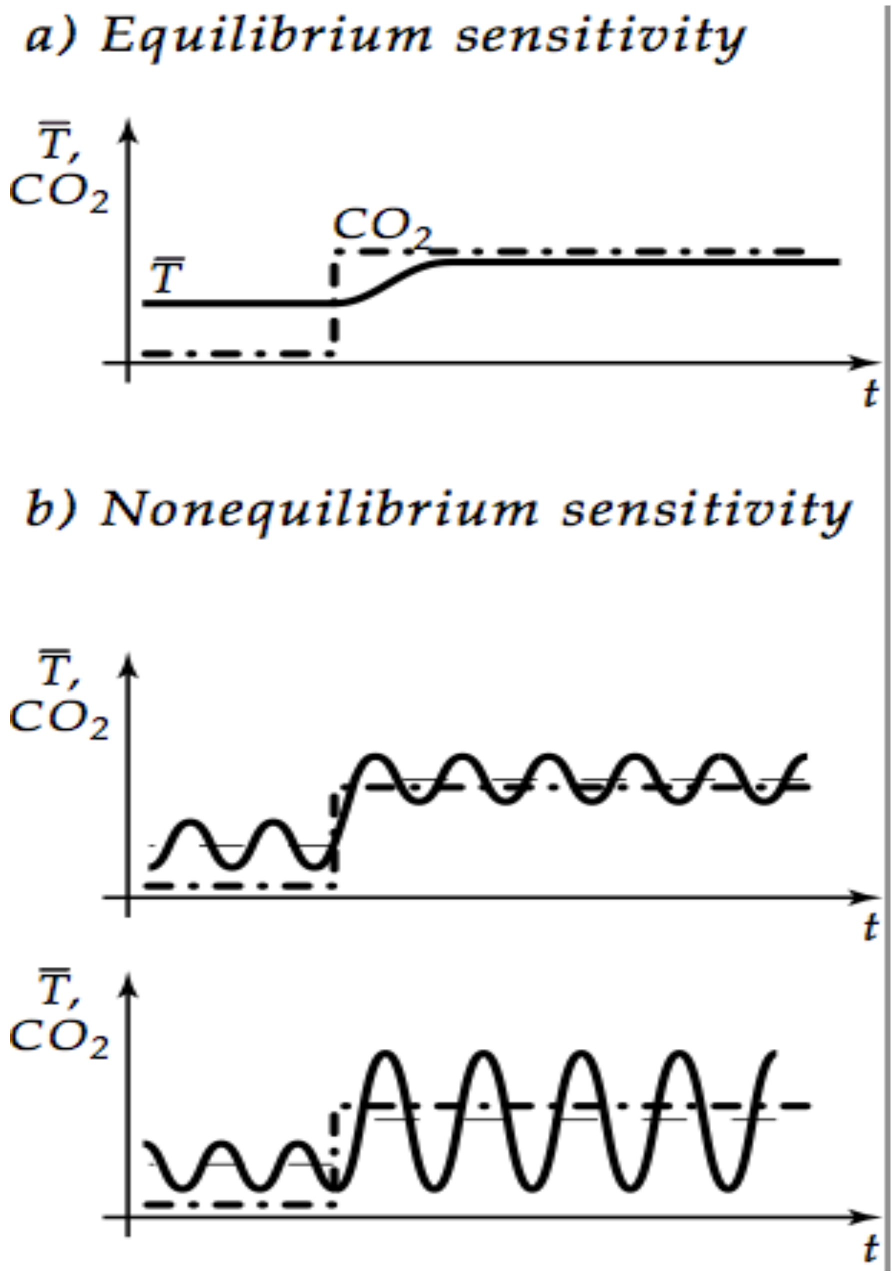
How will "climate" change?

1. Climate is in stable equilibrium (fixed point); if so, mean temperature will just shift gradually to its new equilibrium value.

2. Climate is purely periodic; if so, mean temperature will (maybe) shift gradually to its new equilibrium value. But how will the period, amplitude and phase of the limit cycle change?

3. And how about some "real stuff" now: chaotic + random?

Ghil (Encycl. Global Environmental Change, 2002)



Property of μ_ω for chaotic stochastic systems-I

The Sinai-Ruelle-Bowen (SRB) property

- RDS theory offers a rigorous way to define random versions of stable and unstable manifolds, via the Lyapunov spectrum, the Oseledec multiplicative theorem, and a random version of the Hartman-Grobman theorem.
- When the sample measures μ_ω of an RDS have absolutely continuous conditional measures on the random unstable manifolds, then μ_ω is called a *random SRB measure*.
- If the sample measure of an RDS φ is SRB, then its a “physical" measure in the sense that:

$$\lim_{s \to -\infty} \frac{1}{t-s} \int_s^t G \circ \varphi(s, \theta_{-s}\omega)x \, \mathrm{d}s = \int_{\mathcal{A}(\theta_t\omega)} G(x)\mu_{\theta_t\omega}(\mathrm{d}x), \qquad (3)$$

  for almost every $x \in X$ (in the Lebesgue sense), and for every continuous observable $G : X \to \mathbb{R}$.
- The measure μ_ω is also the image of the Lebesgue measure under the stochastic flow φ: for each region of $\mathcal{A}(\omega)$, it gives the probability to end up on that region, when starting from a volume.

Property of μ_ω for chaotic stochastic systems-II

A remarkable theorem of Ledrappier and Young (1988)

- Ledrappier and Young have proved that, that if the stationary solution, ρ, of the Fokker-Planck equation associated to an SDE presenting a Lyapunov exponent > 0, has a density w.r.t. the Lebesgue measure, then:

  μ_ω is a random SRB measure.

- The domain of application of this theorem is fairly general and shows that a large class of stochastic systems exhibiting a Lyapunov exponent > 0, support a random SRB measure.
- Furthermore, we have the important relation:

$$\mathbb{E}(\mu_\bullet) = \rho, \tag{4}$$

  the stationary solution of the Fokker-Planck equation, when this last one is unique.

The Ruelle response formula

- Physically, the challenge is to find the trade-off between the physics present in the model and the stochastic parameterizations of the missing physics.
  From a mathematical point of view, climate sensitivity could be related to sensitivity of SRB measures.
- The thermodynamic formalism *à la* Ruelle, in the RDS context, helps to understand the response of systems out-of-equilibrium, to changes in the parameterizations (Kifer, Liu, Gundlach).
- The Ruelle response formula: Given an SRB measure μ of an autonomous chaotic system $\dot{x} = f(x)$, an observable $G : X \to \mathbb{R}$, and a smooth time-dependent perturbation X_t, then the time-dependent variations $\delta_t \mu$, of μ is given by:

$$\delta_t \mu(G) = \int_{-\infty}^{t} d\tau \int \mu(dx) X_\tau(x) \cdot \nabla_x (G \circ \varphi_{t-\tau}(x)),$$

where φ_t is the flow of the unperturbed system $\dot{x} = f(x)$.

The susceptibility function

- In the case $X_t(x) = \phi(t)X(x)$, the Ruelle response formula can be written:

$$\delta_t\mu(G) = \int dt'\kappa(t-t')\phi(t'),$$

where κ is called the response function. The Fourier transform $\hat{\kappa}$ of the response function is called the susceptibility function.

- In this case $\widehat{\delta_t\mu(G)}(\xi) = \hat{\kappa}(\xi)\hat{\phi}(\xi)$ and since the r.h.s. is a product, there are no frequencies in the linear response that are not present in the signal.
- In general, the situation can be more complicated and the theory gives the following criteria of high-sensitivity:

  $\mathfrak{C}$: **Poles of the susceptibility function $\hat{\kappa}(\xi)$ in the upper-half plane $\Rightarrow$ High sensitivity of the systems response function $\kappa(t)$.**

- RDS theory offers a path for extending this criteria when random perturbations are considered.

Outline

- The IPCC process: results and further questions.
- Natural climate variability as a source of uncertainties
  - sensitivity to initial data ➔ error growth
  - sensitivity to model formulation ➔ see below!
- Uncertainties and how to fix them
  - structural in/stability
  - random dynamical systems (RDS)
- Two or more illustrative examples
  - Arnol'd tongues and a "French garden"
  - the Lorenz system
  - an ENSO "toy" model
- Linear response theory and climate sensitivity
- Conclusions, work in progress and references

Concluding remarks, I – RDS and RAs

Summary

- A change of paradigm for open, non-autonomous systems
- Random attractors are (i) spectacular, (ii) useful, and (iii) just starting to be explored for climate applications.

Work in progress

- Study the effect of specific stochastic parametrizations on model robustness.
- Applications to intermediate models and GCMs.
- Implications for climate sensitivity.
- Implications for predictability?

Concluding remarks, II – General

What do we know?

- It's getting warmer.
- We do contribute to it.
- So we should act as best we know and can!

What do we know less well?

- By how much?
  - Is it getting warmer …
  - Do we contribute to it …
- How does the climate system (atmosphere, ocean, ice, etc.) really work?
- How does natural variability interact with anthropogenic forcing?

What to do?

- Better understand the system and its forcings.
- Explore the models', and the system's, robustness and sensitivity
  - stochastic structural and statistical stability!
  - linear response = response function + susceptibility function

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Some general references

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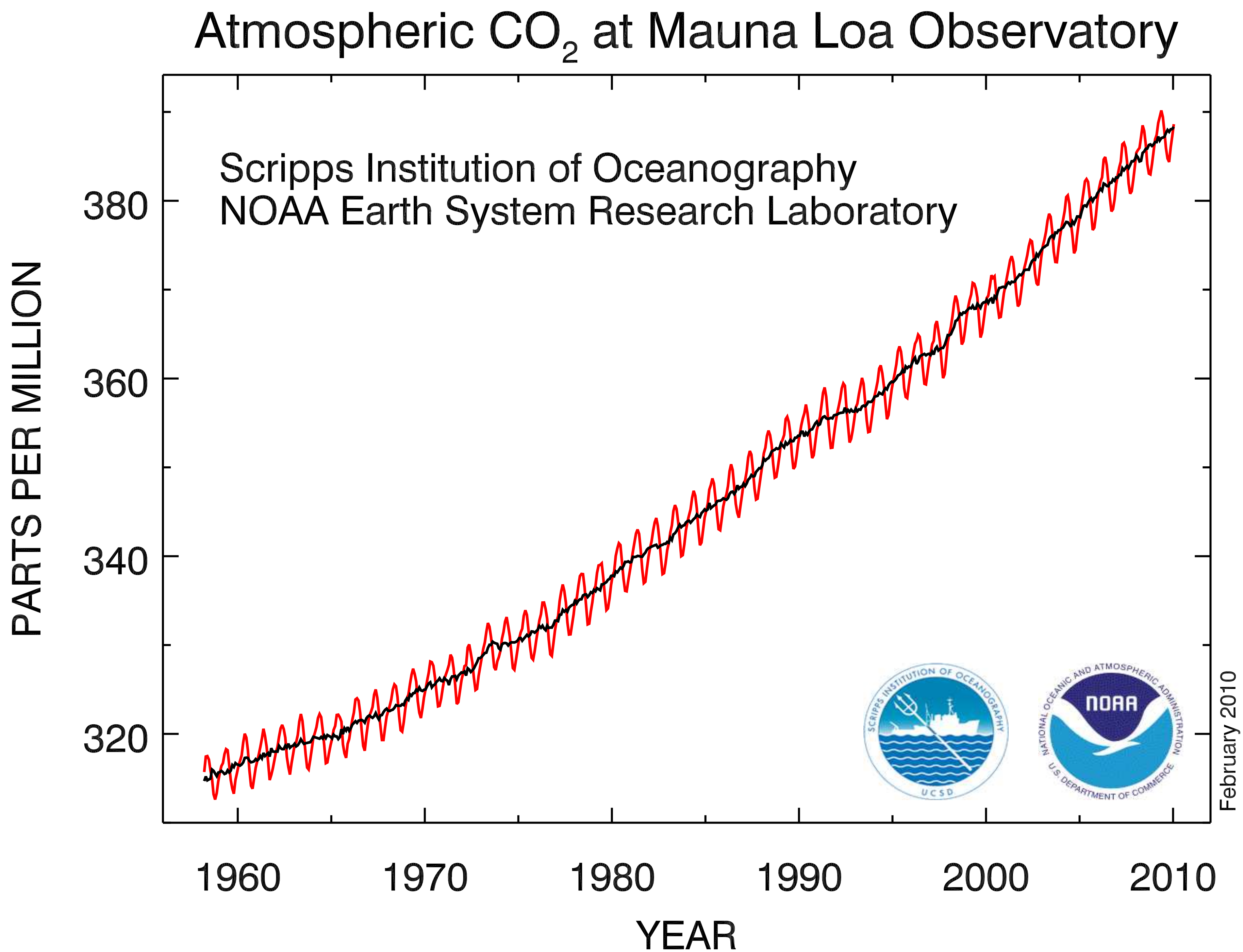
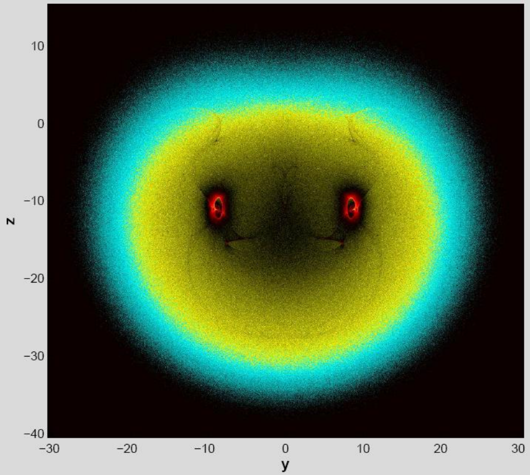
Reserve slides

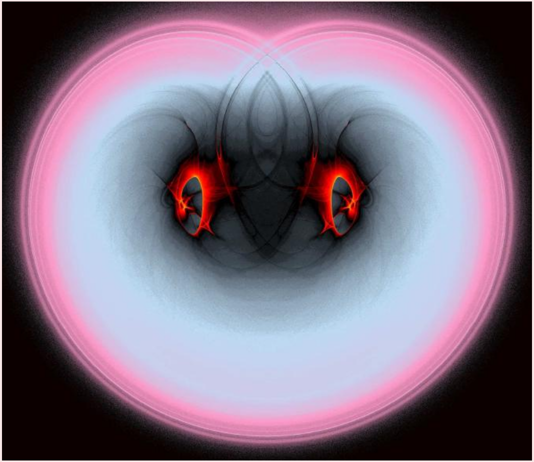

Atmospheric CO_2 at Mauna Loa Observatory
Scripps Institution of Oceanography
NOAA Earth System Research Laboratory
PARTS PER MILLION
320
340
360
380
1960
1970
1980
1990
2000
2010
YEAR
February 2010

Another proj. of the disintegrated measure, more “friendly"



- The next slides are similar, with different noise level α and more I.D....

Disintegration of the measure supported by the R.A.



- 1 Billion I.D., and a different color palette!
- Intensity is $\alpha = 0.2$.
- Do you want different noise intensities?

Climatic uncertainties & moral dilemmas

Thought leaders *Rice, top left, spoke of multilateralism, while Bono, left, demanded more action on poverty. Presidents Karzai and Musharraf, right, both face troubles at home*

♥ ... *keep today's climate for tomorrow?*

Agitator *Gore wants a global compact to tackle climate change and poverty*

♥ *Feed the world today*

or...

Davos, Feb. 2008, photos by *TIME Magazine*, 11 Feb. '08;

see also Hillerbrand & Ghil, *Physica D*, 2008, **237**, 2132–2138, doi:10.1016/j.physd.2008.02.015 .

The Biofuel Myth

- Fine illustration of the moral dilemmas (*).
- Conclusion: "**festina lentae**," as the Romans (**) used to say..

(**) ~ Han dynasty

(*) Hillerbrand & Ghil, *Physica D*, 2008, doi:10.1016/j.physd.2008.02.015, available on line.

Climate Change 1816–2008

M. Gillot, 2008,
Le Monde

T. Géricault, 1819,
Le Louvre