The model of J. C. Yoccoz-H. Birkeland for the populational evolution of the Microtus Epiroticus.

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joint with J. J. Nieto and J. L. Vieitez

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We are interested in the populational evolution of a family of rodents, whose scientific name is "Microtus Epiroticus". These rodents were introduced in the Svaldarb island, Norway, between 1930 and 1960, from Russia (N. Yoccoz),

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♣ How to understand??

Microtus Epiroticus

Figure: The rodent.

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Following these rules, the initial number N of individuals can be estimated.

Distribution of the population by year

Figure: Individuals by year.

Model by J-C. Yoccoz-H. Birkeland

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★ the seasonal factor $m_\rho(t)$ varies sharply from 0 in the winter to 1 from spring through fall. Thus, we take $\rho = 0.30$ and adopt to $m_\rho(t)$

$$m_\rho(t) = \begin{cases} 0 & \text{if } 0 \leq t \mod (1) < \rho \\ 1 & \text{if } \rho \leq t \mod (1) < 1 \end{cases}$$

★ the annual rate of individual reproduction $m(N)$ is very high when $N(t)$ is small: order $m_0 \gtrapprox 30$ and $m(N(t))$ decays abruptly when $N(t)$ grows. Following [Ar], we define $m(N)$ by

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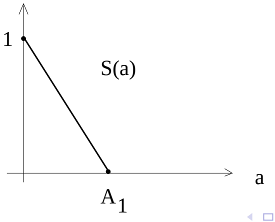

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$$S(j) = \begin{cases} 1 - \frac{j}{2p+1}, & \text{for} \quad j = 0, 1, 2 \ldots, 2p \\ 0 & \text{for} \quad j < 0 \text{ or } \quad j > 2p \end{cases},$$

where $A_1 p = 2p$.

Evolution law

Given an initial vector $(N_0, N_1, N_2, \ldots, N_{2p-1}, N_{2p}) \in \mathbb{R}^{2p+1}$, the evolution of $N(t) = N_t$, $t \in \mathbb{Z}$, is governed by

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$$N_t = \sum_{h=A_0\,p}^{A_1\,p-1} N_{t-h} m(N_{t-h}) m_\rho(t-h) S(h) \Delta h \tag{2}$$

$$= \frac{1}{p} \sum_{h=A_0\,p}^{A_1\,p-1} N_{t-h} m(N_{t-h}) m_\rho(t-h) S(h)\,.$$

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- Recursively we can compute N_j for all $j \geq 0$.

The dynamical system.

Equation (2) defines a discrete dynamical system

$$T : \mathbb{R}^{2p+1} \to \mathbb{R}^{2p+1}$$

$$(N_0, N_1, \ldots, N_{2p}) \mapsto T(N_0, N_1, \ldots, N_{2p}) = (N_p, N_{p+1}, \ldots, N_{3p})$$

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(4) The map $DT(p)$ is non singular.

Upper bound to T

Under certain reasonable conditions, for all $t = 1, 2, \ldots, A_0 p$, it holds

$$N_t \leq N_{max} := m_0 \left(\frac{(A_1 - A_0)^2}{2A_1} \right).$$

When $A_1 = 2$, $A_0 = 0.18$, $m_0 = 50$, $N_{max} \approx 41.4$.

Permanence

A system $T(t)$ is *permanent* if for every positive initial T_0, $\exists\ \epsilon > 0$ s. t. $T(t)$ satisfies

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The system given by equation

$$N_t = \sum_{h=A_0\,p}^{A_1\,p-1} N_{t-h} m(N_{t-h}) m_\rho(t-h) S(h) \Delta h$$

is permanent.

Existence of a compact positive invariant

♠ There is $t_0 > 0$ such that, for all $t \geq t_0$

$$\frac{c_0 m_0}{2} N_{max}^{1-\gamma} \ \leq \ N_t \ \leq \ N_{max}.$$

In particular, T takes the compact set

$$\mathcal{K} = \left[\frac{c_0 m_0}{2} N_{max}^{1-\gamma}, N_{max}\right]^{pA_1+1} \qquad \text{on itself}.$$

Lipschitz

Let H be the set of positive vectors

$$H = \left\{ N = (N_0, N_1, \ldots, N_{2p}) \in \mathbb{R}^{2p+1} | \,\forall\, j : \, N_j > 0 \right\} \,.$$

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$\boxed{T : H \to H \text{ is Lipschitz.}}$

$\boxed{\text{(Fixed point) There exists } p \in \mathcal{K} \text{ such that } T(p) = p.}$

Existence of an attractor Λ

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$\boxed{\text{The basin of attraction } B^s(\Lambda) \text{ of } \Lambda \text{ is simply connected.}}$

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$$\boxed{S(2p-1) > 0 \Longrightarrow DT(x) \text{ is non singular.}}$$

♠ What is the meaning that the probability of survival $S(2p-1)$ is positive?

We interpret that as: the older generations in the vector $N = (N_0, N_1, \ldots, N_{A_1 p})$ has an effective influence in the next population vector $T(N)$, that represents the next geration of Microtus Epiroticus. .

Now comes the numerical approuch.

For the parameters studied by S. Arlot: $A_0 = 0.18$, $\rho = 0.30$, $\gamma = 8.25$:

$$\boxed{\text{the fixed point } p \text{ of } T \text{ is hyperbolic.}}$$

A_0: age of maturity of a female
ρ: size of winter, appears in the sazonal factor m_ρ
γ: appears at the sazonal factor

$$m_\rho(t) = \begin{cases} 0 & \text{if } 0 \leq t \mod (1) < \rho \\ 1 & \text{if } \rho \leq t \mod (1) < 1 \end{cases}$$

Numerical approuch.

★ Assuming that the calculations made by Arlot are sufficiently precise, the eigenvalues $\lambda_i, 1 \leq i \leq 2p$, and μ of $DT(p)$ satisfy $|\lambda_j| << 1$ for all $j = 1, \ldots, 2p$, and $\mu < 0,\ |\mu| > 1$.

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$$\boxed{p \text{ is hyperbolic } \& \ W^s(p) \text{ is codimension one.}}$$

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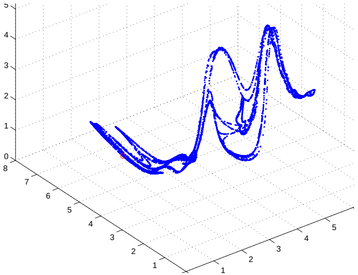

Figure: The attractor

The closure of $W^u(p)$

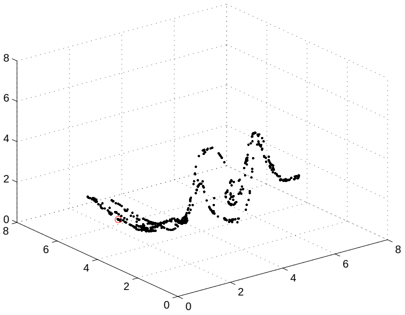

Figure: $\overline{W^u(p)}$

When $\overline{W^u(p)} = \Lambda$

$\boxed{\text{If there is } q \in W^u(p) \text{ such that } \overline{W^u(q)} = \Lambda}$

$$\Downarrow$$

$\boxed{\Lambda \text{ is topologically mixing}}$

Sensitiveness with respect to initial data.

$f : K \to K$ is sensitive with respect to initial conditions if $\exists$ $\alpha > 0$ such that, $\forall\ x \in K$ and $\forall$ neighborhood $U(x)$, $\exists\ y \in U(x)$ and $n \in I\!\!N$ such that

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Sensitiveness with respect to initial data.

$f : K \to K$ is sensitive with respect to initial conditions if $\exists$ $\alpha > 0$ such that, $\forall\ x \in K$ and $\forall$ neighborhood $U(x)$, $\exists\ y \in U(x)$ and $n \in I\!N$ such that

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Thus, it is not possible to predict the behavior of the evolution of the population.

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The Kolmogorov entropy of an attractor can be considered as a measure for the rate of information loss along the attractor, or as a measure for the degree of predictability of points along the attractor given an initial data.

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In general, a **positive entropy** is considered as a conclusive proof that the dynamical system is **chaotic**. A **zero entropy** represents a constant or a regular phenomena that can be represented by a fixed point or a **periodic attractor**.

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$\boxed{\text{From the data, } K \simeq 0.37.}$

Existence of a homoclinic point

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To verify this fact we proceed as follows:

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We search the fixed point whose existence was already proved.

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★ Chose a certain group G of points $p_j, 0 \leq j \leq m$, and search in the simplex $S = \{b_0 p_1 + b_1 p_2 + \cdots + b_m p_m, \sum_i b_i = 1, p_j \in Q\}$ those points minimizing $\mathrm{dist(x, T^2(x))}$, $x \in S$.

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Thus, p is a hyperbolic periodic point with codimension 1 stable manifold.

The aproximated unstable manifold

★ Since the eigenvalue μ is negative, the iterates $T^{2k}(p)$ and $T^{2(k+1)}(p)$ alternate with respect to $W^s(p)$.

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$T^{2n}(L) = W$, $n = 629$, is a good aproximated unstable manifold.

The aproximated unstable manifold

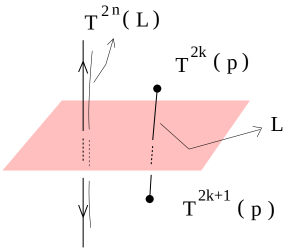

Figure: $T^{2n}(L) = W \approx W^u(p)$

From the data we obtain that the length of $T^{2n}(L) = W$ is $\approx 5.10^{-4}$.

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★ We take this iterate $(T^2)^{629}(e_j)$ as a good candidate and we denote it by E.

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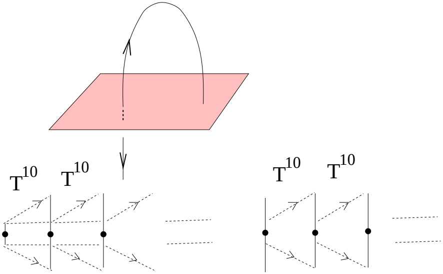
★ Denote $T^{10}(I_0)$ by I_1, repeat up to the 1258-iterate.



Transverse

★ Denote by I_{m_0} this last iterate and write $I_{m_0} = [r, l]$.
Observe that $(E = T^2)^{629}(e_0) \in I_{m_0}$.
Consider the vectors: $V_r = \vec{pr}$ and $V_l = \vec{pl}$.

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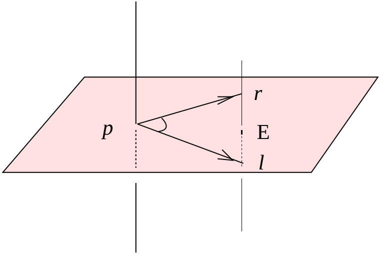

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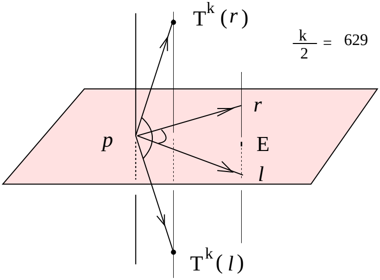

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★ the entropy of the system is positive

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Thus, numerically evidence that the system is chaotic.

A homoclinic point in Λ-I.

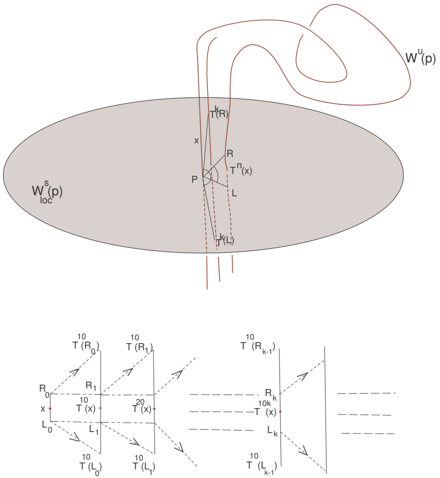

Figure:

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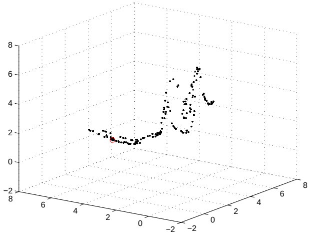

Figure: Iterates of a small segment at the homoclinic point

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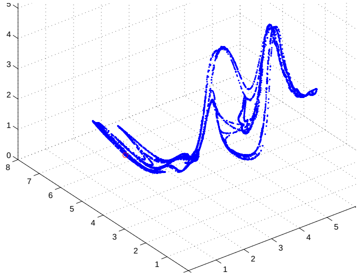

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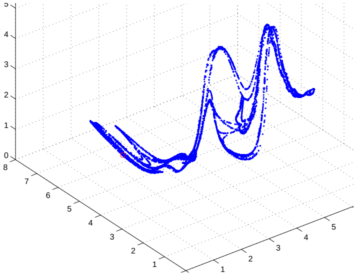

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♠ $T/B^s(\Lambda)$ is a diffeo and there is a fixed point $p \in \Lambda$ such that $DT(p)$ has a *unique negative real* eigenvalue μ with $|\mu| > 1$ and all the other satisfies $0 < |\lambda_i| << 1$. Thus, there is a a neighborhood U, $p \in U$, such that T/U is sectionally dissipative.

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Conclusion

These analysis suggest that any “robustness” of the attractor, has to hold in the framework of non uniform hyperbolicity.

Thus, the propose to future work is:

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Since $\dim(E^u) = 1$, on the light of the Henon attractor, we would like to obtain a 1-dimensional dynamics, adding hipotheses on the probability of surviving $S(a)$: instead of decreasing linearly, decreasing super fast.

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End

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